## EXAMINATION OF THE INFLUENCE OF SHEAR MICRO-GEOMETRICAL PROPERTIES ON TRANSVERSE ELASTICITY THE MODULUS OF ROVING COMPOSITE MATERIALS USED IN CRITICAL CONSTRUCTIONS

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### ABSTRACT

Monitoring of quality is an important task in the production process of critical supporting structures made from roving composites. Results from the research performed on the production samples of girders of helicopter blades presented in this paper, suggest that geometrical measures can be used for prediction of material strength and durability. The correlation between geometrical measurements of shear microstructure and values of the elasticity modulus determined in the transverse shear strength test by short beam method was assessed.

## 1. INTRODUCTION

Composites made up of fibres (glass or carbon) with a circular cross section, arranged unidirectionally and embedded in an epoxy resin matrix constitute an interesting group among machine building materials. The composite of hard, brittle fibres immersed in an epoxy resin of significantly lower strength is strongly anisotropic. Strength of the composite depends on: the direction of load in relation to the fibre axis, the strength of the components, the adhesive forces between the fibres and the resin, on the relative volume of the components and on the geometrical arrangement of fibres in a cross-section. The geometrical arrangement of fibres in a cross-section.

One of the main problems in material strength analysis is to determine the geometrical distribution of fibres in a sample and obtain a measure that can be related to material strength. A literature review assumes that the geometric properties of the microstructure are random [4]. However, in theoretical considerations of composite strength, the following assumptions are made [3]: composite components are homogeneous (there are no defects such as bubbles and impurities in the matrix volume), component materials are isotropic (anisotropy of the fibres is ignored), elastic deformation of composite ingredients is linear, a combination of fibres and matrix is ideal and the strength of the adhesion exceeds the strength of the warp (no slip and

tear), the cross-section of fibres is circular or rectangular (can be approximated with basic geometrical shapes like circle or rectangle).

There are additional simplification of the models, such as: all fibres are uniform in diameter (can be represented as points in geometrical distribution models) and there is no fibres-to-matrix adhesion (thickness of the interface is ignored). The results quoted in the literature concern models, rather than real objects which are likely be characterized by considerable randomness [7].

A standard evaluation of the geometrical distribution of composite ingredients is made on the basis of an image of the microstructure. The evaluation is made from: number, size, shape and position of every component [5, 6]. Various local and global indicators are calculated from these values.

The following methods for characterizing heterogeneity of the structure of multiphase materials are mentioned [7, 8, 9, 10, 11, 12, 13]:

- systematically arranged test elements (secants) like: systematic scanning (grid fields, open curvilinear shapes), covariance function (parallel secants), radial distribution function (circular test elements),
- distance and angle between the neighbouring fibres,
- properties of the tessellation polygons, including: field  $(A_i)$ , circuit  $(B_i)$  and number of polygon sides, the thickness of the matrix between neighbouring fibres and the aspect ratio,
- topological entropy and functions for local concentrations.

# 2. THE STUDY MATERIALS AND METHODS

The composites studied were made from glass fibres and epoxy resin. The samples, obtained from the production process of helicopter blades, were tested for endurance in a laboratory and then cross-sections were made for the geometrical analysis.

Laboratory tests according to the PN-EN ISO 14130:2001 (determination of the shear strength by a short-beam method) and BS-EN-ISO-14125:2001 (bending strength test) were performed on the samples. Additionally, the results of flexural strength tests according to PN-79/C-89027, performed by the manufacturer were collected. Bitmap images of the cross-sections were used for geometrical analysis. The experimental material consisted of over 1400 bitmap images taken from 81 composite samples.

As the results of the geometrical analysis of bitmap images, the following were obtained: parametrized microstructural images, fibre coordinates and their diameters, defect locations and surfaces (inclusions and gas pores). The relative volume of glass and gas pores, as well as measures characterizing the distribution of fibres were then calculated from these sets. All measurements and calculations were performed using the computerized vision system developed by the author [14]. Sample results are shown in Fig. 1.



Fig. 1 a) Bitmap images of cross-sections of composite samples, parametrized microstructural images with tessellation polygons. Topological entropy values a) S = 1.59, b) S = 1.06 (author's study)

As one of the main factors determining the composite properties is the area of contact between components and the strength of adhesive forces, it seems appropriate to measure the fibre distribution homogeneity. Thus a measure of matrix film thickness was proposed, similar to those documented in [15, 16, 17, 18].

The measurements characterizing composite macro-structure and design assumptions such as the relative volume of the glass Usz [%] and fibre diameter - Dw [ $\mu$ m] were considered during the research. Additionally, the relative volume of the gas pores [%] and fibre arrangements defined by the topological entropy *S* and the minimum distance between neighbouring fibres  $G_{Imin}$  [ $\mu$ m] were calculated.

As shown in Fig. 2, the matrix film thickness around a single fibre can be defined by  $G_1$ ,  $G_2$ ,  $G_{AB}$ ,  $G_{AD}[\mu m]$ , where:

 $G_1$  – half of the distance between neighbouring fibres,

 $G_2$  – local film thickness measured at regular intervals along the perimeter,

 $G_{AD}$  – the average value of film thickness calculated by the formula (1):

$$G_{AD} = \frac{\sqrt{D_w^2 + 4\frac{A_o}{\pi}} - D_w}{2}, \qquad (1)$$

 $G_{AB}$  – the proportion of matrix area in the tessellation polygon  $A_o$  to the fibre circuit  $B_w$ , calculated according to the formula (2):

$$G_{AB} = \frac{A_o}{\pi D_w} \tag{2}$$

The geometrical measurements documented in the literature like tessellation polygon properties (number of sides, circumference and the area of the polygon) and the ratio of the area of the tessellation polygon and the fibre cross-section (related to the relative volume of the glass) [13]

are illustrated in Fig 2.



Fig. 2 a) Local thickness of the polygon layer around a single fibre, b) ring model of the matrix around fibre (author's study)

Fig. 3 presents a fragment of a geometrical model, determined for a sample composite specimen, with average matrix film thickness measures  $G_I$ ,  $G_I$ , and  $G_{AD}$  represented by the circles coaxial with the corresponding fibre.



Fig. 3: Fragment of roving composite microstructural model with circles significant averages of matrix layers around singles fibres  $\overline{G}_1, \overline{G}_2, G_{AB}$  and  $G_{AD}$  (author's study) [19]

Fig. 4 illustrates the how the local film thickness measured around the fibre perimeter (dashed curve) relates to the values of the corresponding average matrix film thickness measures  $G_{I}$ ,  $G_{I}$ , and  $G_{AD}$  calculated for the fibre.



Fig. 4: Thickness of the matrix layer circumference single fibre along its perimeter (horizontal axis - angle +/-180°) defined as a measurements:  $G_1, \overline{G}_1, G_2, \overline{G}_2, G_{AB}$  (vertical axis) (author's study) [19]

# 3. INTERPRETATION OF THE RESULTS OBTAINED

The set of material strength characteristics contained 11 measurements, while the set of geometrical microstructural measurements had 13 values for each sample. In the final phase of the statistical correlation analysis, the geometrical measurements were tested for correlation with the material strength values obtained form shear strength tests [20].

The uniform high quality of the samples received, resulting from the quality control level and manufacturing standards in the aerospace industry made it difficult to determine a strong correlation between geometrical measures and strength test results. However, a comparison of two sets of samples, one containing the highest value of transverse elasticity modulus  $E_{2t}$ , (set A) and the other, containing the lowest, located on opposite sides of the  $E_{2t}$  range (set B), shows a correlation between the strength of the sample and its geometrical properties. The differentiation between the sets A and B, confirmed by a Student's t-test is illustrated in Fig. 5.



Fig. 5: Modulus of shear averaged values  $(\Box)$  in two groups of samples, raw data  $(\Delta)$  and

confidence interval  $(\frac{1}{T})$  (author's study)

Thus, there is a problem requiring explanation: does any of the geometrical measures correlate with the strength measurements. In the first instance the fibre diameter  $D_w$ , was analysed.

Fig. 6 shows distributions of fibre diameters  $D_w[\mu m]$  in both groups of samples. The difference between average values of  $D_{w(A)}$  and  $D_{w(B)}$  is significantly lower than the variation  $\delta_{w(A)}$  and  $\delta_{w(B)}$  in both groups. That similarity indicates that no significant correlation exists between the fibre diameter and sample strength.



Fig. 6: Fibre diameter  $D_w$  distribution: a) set A, b) set B (author's study)

Another quantity tested for correlation was the relative volume of glass  $U_{sz}$  [%]. Fig. 7 shows the distributions of the relative volume of glass  $U_{sz}$  [%] in both sets. It was found that the average values in both sets differ, what may suggest the existence of a correlation.



Fig. 7: Distributions of glass relative volume: a) set A, b) set B (author's study)

A discriminant function an analysis was used to determine the importance of the selected factors [21].

On the basis of the correlation analysis three of the total group of 13 measures were selected:

- $G_{AB}$  conventional matrix film thickness around a single fibre,
- $G_{1min}$  the minimum matrix thickness around a single fibre,
- $U_{sz}$  the glass relative volume of glass.

Locations of the members of the sets A and B in the space defined by variables selected in the correlation analysis  $[U_{sz}, G_{AB}, G_{1min}]$  are shown in Fig 8.



Fig. 8:  $U_{sz}$ ,  $G_{AB}$  i  $G_{1min}$  values of subsets A( $\circ$ ) and B( $\Box$ ) samples (author's study)

A discriminant function for this case could be defined as:

$$D_1 = 14,357 + 1,649 G_{AB} - 16,944 G_{1min} - 0,396 U_{sz}.$$
 (3)

Its significance level, as defined in a chi-square test, was p = 0.0001. The high value of the canonical correlation R = 0.9151 indicates a strong correlation between groups and the discriminant function [22]. Consequently, the function (3) is a good classifier for the sets A and B.

The results of the above analysis were verified by applying the discriminant function (3) to all samples. In the results, all but one of the original members of the sets A and B were classified incorrectly (93.3% accuracy).

A classification based on an evaluation of distances from individual cases to block centroids and a probability analysis of specimens classifications to appropriate groups were consistent with the results obtained by using the classification function.

The coefficients in the formula (3) were evaluated from the standardized coefficients of the discriminant function (Table 1), and structural factor coefficients (Table 2) [21].

Table 1: Discriminant function coefficients (author's study)

Coefficients	
Raw	Standardized

Constant	14,357	
G <sub>AB</sub>	1,649	0,448
<b>G</b> <sub>1min</sub>	-16,944	-0,746
U <sub>sz</sub>	-0,396	-0,945
Eigenvalue	5,151	

The standardized coefficients of the discriminant function determine how the particular variable contributes to the group differentiation into the sets A and B. The highest standardized coefficient value in Table 1 reaches the  $U_{sz}$  characteristic, which is -0.945. It is worth noting, that the quality  $G_{AB}$  is related to  $U_{sz}$  [18]. Additionally, the coefficient of  $G_{AB}$  is inversely proportional to  $U_{sz}$ , which means that the contribution made by of one characteristic is decreased by the influence of the other. In this situation, a structural factor analysis was performed to find out the real influence of both factors (Table 2).

Table 2: Geometrical characteristics – canonical coefficient correlation (author's study)

	Element 1
<b>G</b> <sub>AB</sub>	0,837
<b>G</b> <sub>1min</sub>	0,188
U <sub>sz</sub>	-0,810

Structural factor coefficients (Table 2) determine the individual contribution of characteristics to the value of the discriminant function [21, 22]. Characteristics  $G_{AB}$  and  $U_{sz}$  have similar, high values of the coefficients (Table 2), while the correlation coefficient of  $G_{AB}$  is greater than the coefficient of  $U_{sz}$ .

As the results show, the fibre diameter quality has no significant influence on sample strength. This confirms the information contained in the histogram shown in Fig. 6. in accordance with the plan for obtaining specimens, with the same fibre diameter. While an essential influence of the relative volume of the glass  $U_{sz}$  on the variability in this group of specimens makes it difficult to analyse the effects of the fibre arrangements.

The results of the discriminant function analysis (performed on sets A and B) proved as significant the following geometrical characteristics:  $G_{AB}$ ,  $G_{Imin}$ ,  $U_{sz}$ . Therefore, a verification of these characteristics impact on the value of the transverse elasticity modulus  $E_{2t}$  was made. The verification was carried out using a multiple regression analysis method.

The aim of this analysis was to determine whether the characteristics selected during the discriminant analysis clearly and thoroughly define the composite structure, and if the transverse elasticity modulus value  $E_{2t}$  could be determined from these characteristics.

The aim of the analysis is to find the coefficients of the function  $E_{2t}(G_{1min}, G_{AB}, U_{sz})$ , and their interpretation.

A linear regression equation obtained from the analysis has the following form:

$$E_{2t} = 93,82 - 20,52 G_{AB} + 77,74 G_{Imin} - 0,0076 U_{sz} \pm 6,47$$
(4)

Influence of shear properties on transverse modulus critical constructions

(115,4) (13,7) (50,9) (1,7)

Below the equation, in parentheses standard errors for estimating the value  $E_{2t}$  are listed. The resulting model explains a 74.39% variation of  $E_{2t}$  in the considered specimen group.

Analysing the relationship (4) it should be noted that in the examined variability of the research material, the greatest impact on the value of  $E_{2t}$  was the average matrix thickness around fibre  $G_{AB}$ .

Given that the characteristics of composite construction are interlinked, the sense of the coefficients in the linear equation is easy to interpret and consistent with observations made during the discriminant analysis.

It should also be noted that the value of the average matrix thickness around the fibres (for example, expressed by measure  $G_{AB}$ ) is related to the relative volume of glass. However, the  $G_{AB}$  value may be increased if there are gas pores in the cross-section surface. The presence of gas pores reduces the value of the transverse elasticity modulus  $E_{2t}$ . Increasing the minimum distance between neighbouring fibres  $G_{1min}$  will improve the composite elasticity (a lower number of adjoining fibre pairs). The influence of the relative volume of the glass  $U_{sz}$  on  $E_{2t}$  is not significant in relation to other characteristics. This is probably due to, the fact that the relative volume of glass is already included in  $G_{AB}$ .



Fig. 9: Predicted and observed values  $E_{2t}$  correlation (author's study)

The results of the transverse elasticity modulus  $E_{2t}$  calculated from equation (2) and compared to the strength test results are shown in Fig 9. Most of the cases are located within the confidence limits. These results are consistent with the relation (3).

# 4. CONCLUSIONS

It has been shown that the strength of a roving composite defined as the transverse elasticity modulus value  $E_{2t}$ , in the range of variability of the samples examined could be classified in different groups by using the results of the geometrical analysis of the cross-section: the average matrix thickness around a fibre  $\overline{G}_{AB}$ , the minimum thickness of the matrix  $G_{1min}$  and the relative volume of the glass  $U_{sz}$ . In the range of characteristics considered these quantities form a linear equation which makes it possible to predict the transverse elasticity modulus  $E_{2t}$ .

The most significant contribution to the transverse elasticity modulus comes from the average matrix thickness around a single fibre  $\overline{G}_{AB}$ .

A correlation of the micro-geometrical properties of the composite construction, expressed mainly by the  $G_{AB}$  to its mechanical properties (transversal elasticity modulus) was proved experimentally. The composite material should not be treated as homogeneous in cases of cross loading and strength calculations should take into account the possible variability of the average matrix thickness around a single fibre  $G_{AB}$ .

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